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Three Dimensional Finite-Difference Time-Domain Solution of Maxwell's Equations With Perfectly Matched Absorbing Layers

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FOREWORD

This report presents a three-dimensional finite-difference time-domain formulation of Maxwell's equations with the absorbing shell consisting of perfectly matched layers. This work was performed at the Naval Air Warfare Center Weapons Division, China Lake, California, during fiscal year 1999 in support of exploratory development efforts investigating waveguide modeling for the Precision Strike Navigator. Mike Bramson monitored this Office of Naval Research sponsored work under fund document N0001499WX20026.

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(U) We formulate and document Maxwell's equations in finite-difference time-domain form and for three-dimensional geometry. We incorporate the successful perfectly matched layer absorbing boundary conditions and the coupled-equation-set is presented in various forms depending on space points being inside or outside the absorbing shell. The material media are nonpermeable and nondispersive. These equations provide a basis from which problems requiring time-domain solutions to Maxwell's equations may be approached.

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INTRODUCTION

This document outlines a basic framework for time-domain solutions to Maxwell's equations in three-dimensional geometry. The outer limits of the computational domain consists of absorbing layers as described by Berenger (References 1 and 2). The computational domain grid consists of square cells as described by Yee (Reference 3).

MAXWELL'S EQUATIONS

Following Berenger (References 1 and 2), we include anisotropic electric conductivity σ and magnetic conductivity σ^*. We assume that the media is non-permeable and the permittivity ε is non-dispersive. The full set of coupled Maxwell's equations are written as

$$\frac{\varepsilon}{c}\frac{\partial E_{xy}}{\partial t}+\frac{4\pi\sigma_y}{c}E_{xy}=\frac{\partial}{\partial y}\left(H_{zx}+H_{zy}\right) \tag{1}$$

$$\frac{\varepsilon}{c}\frac{\partial E_{xz}}{\partial t}+\frac{4\pi\sigma_z}{c}E_{xz}=-\frac{\partial}{\partial z}\left(H_{yz}+H_{yx}\right) \tag{2}$$

$$\frac{\varepsilon}{c}\frac{\partial E_{yz}}{\partial t}+\frac{4\pi\sigma_z}{c}E_{yz}=\frac{\partial}{\partial z}\left(H_{xy}+H_{xz}\right) \tag{3}$$

$$\frac{\varepsilon}{c}\frac{\partial E_{yx}}{\partial t}+\frac{4\pi\sigma_x}{c}E_{yx}=-\frac{\partial}{\partial x}\left(H_{zx}+H_{zy}\right) \tag{4}$$

$$\frac{\varepsilon}{c}\frac{\partial E_{zx}}{\partial t}+\frac{4\pi\sigma_x}{c}E_{zx}=\frac{\partial}{\partial x}\left(H_{yz}+H_{yx}\right) \tag{5}$$

$$\frac{\varepsilon}{c}\frac{\partial E_{zy}}{\partial t}+\frac{4\pi\sigma_y}{c}E_{zy}=-\frac{\partial}{\partial y}\left(H_{xy}+H_{xz}\right) \tag{6}$$

$$\frac{1}{c}\frac{\partial H_{xy}}{\partial t}+\frac{4\pi\sigma_y^*}{c}H_{xy}=-\frac{\partial}{\partial y}\left(E_{zx}+E_{zy}\right) \tag{7}$$

$$\frac{1}{c}\frac{\partial H_{xz}}{\partial t}+\frac{4\pi\sigma_z^*}{c}H_{xz}=\frac{\partial}{\partial z}\left(E_{yz}+E_{yx}\right) \tag{8}$$

$$\frac{1}{c}\frac{\partial H_{yz}}{\partial t}+\frac{4\pi\sigma_z^*}{c}H_{yz}=-\frac{\partial}{\partial z}\left(E_{xy}+E_{xz}\right) \tag{9}$$

$$\frac{1}{c}\frac{\partial H_{yx}}{\partial t}+\frac{4\pi\sigma_x^*}{c}H_{yx}=\frac{\partial}{\partial x}\left(E_{zx}+E_{zy}\right) \tag{10}$$

$$\frac{1}{c}\frac{\partial H_{zx}}{\partial t}+\frac{4\pi\sigma_x^*}{c}H_{zx}=-\frac{\partial}{\partial x}\left(E_{yz}+E_{yx}\right) \tag{11}$$

$$\frac{1}{c}\frac{\partial H_{zy}}{\partial t}+\frac{4\pi\sigma_y^*}{c}H_{zy}=\frac{\partial}{\partial y}\left(E_{xy}+E_{xz}\right) \tag{12}$$

The purpose of anisotropy in the conductivity, and the splitting of the electric and magnetic fields, is to allow the freedom to selectively create absorption along specified directions. Further, these conductivities and the field splitting are non-physical and are introduced only to create a perfectly matched absorbing shell that surrounds the computational domain. The absorbing shell is designed to simulate an infinite sized domain by virtue of the lack of reflected fields.

We now rewrite Equations 1 through 12 by using identities of the following form. Multiply Equation 1 by $\exp\left[4\pi\sigma_y t/\varepsilon\right]$ and we see that

$$\frac{\varepsilon}{c}\frac{\partial}{\partial t}\left[e^{4\pi\sigma_y t/\varepsilon}E_{xy}\right]=e^{4\pi\sigma_y t/\varepsilon}\left[\frac{\varepsilon}{c}\frac{\partial E_{xy}}{\partial t}+\frac{4\pi\sigma_y}{c}E_{xy}\right]$$

Comparing this with Equation 1, we can rewrite Equation 1 (and similarly, Equations 2 through 12) as

$$\frac{\varepsilon}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_y t/\varepsilon\right]E_{xy}\right\}=\exp\left[4\pi\sigma_y t/\varepsilon\right]\frac{\partial}{\partial y}\left\{H_{zx}+H_{zy}\right\} \tag{13}$$

$$\frac{\varepsilon}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_z t/\varepsilon\right]E_{xz}\right\}=-\exp\left[4\pi\sigma_z t/\varepsilon\right]\frac{\partial}{\partial z}\left\{H_{yz}+H_{yx}\right\} \tag{14}$$

$$\frac{\varepsilon}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_z t/\varepsilon\right]E_{yz}\right\}=\exp\left[4\pi\sigma_z t/\varepsilon\right]\frac{\partial}{\partial z}\left\{H_{xy}+H_{xz}\right\} \tag{15}$$

$$\frac{\varepsilon}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_x t/\varepsilon\right]E_{yx}\right\}=-\exp\left[4\pi\sigma_x t/\varepsilon\right]\frac{\partial}{\partial x}\left\{H_{zx}+H_{zy}\right\} \tag{16}$$

$$\frac{\varepsilon}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_x t/\varepsilon\right]E_{zx}\right\}=\exp\left[4\pi\sigma_x t/\varepsilon\right]\frac{\partial}{\partial x}\left\{H_{yz}+H_{yx}\right\} \tag{17}$$

$$\frac{\varepsilon}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_y t/\varepsilon\right]E_{zy}\right\}=-\exp\left[4\pi\sigma_y t/\varepsilon\right]\frac{\partial}{\partial y}\left\{H_{xy}+H_{xz}\right\} \tag{18}$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_y^* t\right]H_{xy}\right\}=-\exp\left[4\pi\sigma_y^* t\right]\frac{\partial}{\partial y}\left\{E_{zx}+E_{zy}\right\} \tag{19}$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_z^* t\right]H_{xz}\right\}=\exp\left[4\pi\sigma_z^* t\right]\frac{\partial}{\partial z}\left\{E_{yz}+E_{yx}\right\} \tag{20}$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_z^* t\right]H_{yz}\right\}=-\exp\left[4\pi\sigma_z^* t\right]\frac{\partial}{\partial z}\left\{E_{xy}+E_{xz}\right\} \tag{21}$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_x^* t\right]H_{yx}\right\}=\exp\left[4\pi\sigma_x^* t\right]\frac{\partial}{\partial x}\left\{E_{zx}+E_{zy}\right\} \tag{22}$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_x^* t\right]H_{zx}\right\} = -\exp\left[4\pi\sigma_x^* t\right]\frac{\partial}{\partial x}\left\{E_{yz} + E_{yx}\right\} \tag{23}$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left\{\exp\left[4\pi\sigma_y^* t\right]H_{zy}\right\} = \exp\left[4\pi\sigma_y^* t\right]\frac{\partial}{\partial y}\left\{E_{xy} + E_{xz}\right\} \tag{24}$$

Equations 13 through 24 represent our coupled set of Maxwell's equations to solve after they are written with finite difference approximations for the space and time derivatives. These equations are explicitly written with $\sigma_j \neq 0$ and $\sigma_j^* \neq 0$ and this occurs only when certain points in the computational domain are in an absorbing boundary layer. In such a region, impedance matching dictates that

$$\frac{\sigma_x}{\varepsilon} = \sigma_x^* \ , \ \frac{\sigma_y}{\varepsilon} = \sigma_y^* \ , \ \frac{\sigma_z}{\varepsilon} = \sigma_z^* \tag{25}$$

In any region of the computational domain that is outside an absorption region, $\sigma_j = \sigma_j^* = 0$, and in their continuous form, this means that Equations 13 through 24 can be combined into the usual form of Maxwell's equations with

$$E_{xy} + E_{xz} = E_x \text{ and } E_{yx} + E_{yz} = E_y \text{ and } E_{zx} + E_{zy} = E_z \tag{26}$$

$$H_{xy} + H_{xz} = H_x \text{ and } H_{yx} + H_{yz} = H_y \text{ and } H_{zx} + H_{zy} = H_z \tag{27}$$

SPACE AND TIME DISCRETIZATION

In order to write Equations 13 through 24 in finite difference form, we let a point in space be represented by (References 3 and 4)

$$(i,j,k) = (i\Delta x, j\Delta y, k\Delta z) \tag{28}$$

and a function of space and time by

$$F^n(i,j,k) = F(i\Delta x, j\Delta y, k\Delta z, n\Delta t) \tag{29}$$

where i, j, k, and n are integers. We want to use centered space and time derivatives to achieve accuracy $O(\Delta x^2)$, etc., and $O(\Delta t^2)$. Notation for space derivatives has the form

$$\frac{\partial F^n(i,j,k)}{\partial x} = \frac{F^n(i+1/2,j,k) - F^n(i-1/2,j,k)}{\Delta x} + O(\Delta x^2) \tag{30}$$

and time derivatives are similarly written

$$\frac{\partial F^n(i,j,k)}{\partial t} = \frac{F^{n+1/2}(i,j,k) - F^{n-1/2}(i,j,k)}{\Delta t} + O(\Delta t^2) \tag{31}$$

A three-dimensional grid composed of square cells will characterize the computational domain. Equations 13 through 24 will be discretized over a three-dimensional Yee cell (Reference 4) as shown in Figure 1. For the case where a space grid point (i, j, k) lies within the absorption shell, Equations 13 through 24 are written differently for the side,

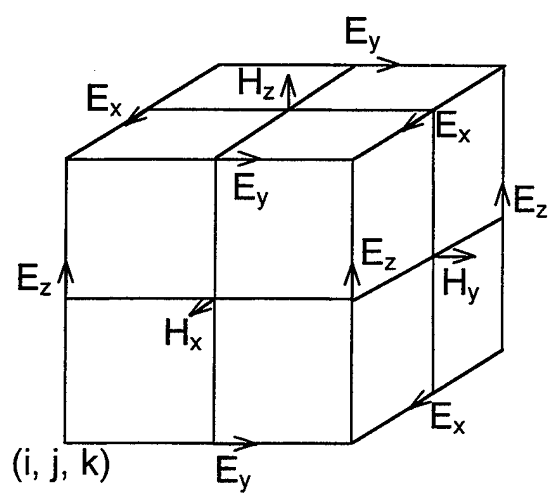

FIGURE 1. Computational Domain Unit Cell in Cartesian Coordinates.

edge, or corner regions of the shell. In a corner region of the absorption shell, we want absorption to occur in all three directions and generally, at a given point in a corner, all three σ_j quantities are non zero and different. In an edge region of the absorption shell, one of the σ_j quantities will be zero since absorption will occur in two of three directions. The remaining two nonzero σ_j quantities will generally be different at a point in an edge. For either the corner or edge region, Maxwell's equations take the general form

FIELD POINT IN CORNER OR EDGE REGION OF THE ABSORPTION SHELL

$$E_{xy}^{n+1}(i+1/2,j,k)=\exp\left(\frac{-4\pi\Delta t\sigma_y(i+1/2,j,k)}{\varepsilon(i+1/2,j,k)}\right)E_{xy}^{n}(i+1/2,j,k)+\frac{c\Delta t}{\Delta y\varepsilon(i+1/2,j,k)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma_y(i+1/2,j,k)}{\varepsilon(i+1/2,j,k)}\right)\Big\{H_{zx}^{n+1/2}(i+1/2,j+1/2,k)-H_{zx}^{n+1/2}(i+1/2,j-1/2,k) \quad (32)$$

$$+H_{zy}^{n+1/2}(i+1/2,j+1/2,k)-H_{zy}^{n+1/2}(i+1/2,j-1/2,k)\Big\}$$

$$E_{xz}^{n+1}(i+1/2,j,k)=\exp\left(\frac{-4\pi\Delta t\sigma_z(i+1/2,j,k)}{\varepsilon(i+1/2,j,k)}\right)E_{xz}^{n}(i+1/2,j,k)-\frac{c\Delta t}{\Delta z\varepsilon(i+1/2,j,k)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma_z(i+1/2,j,k)}{\varepsilon(i+1/2,j,k)}\right)\Big\{H_{yz}^{n+1/2}(i+1/2,j,k+1/2)-H_{yz}^{n+1/2}(i+1/2,j,k-1/2) \quad (33)$$

$$+H_{yx}^{n+1/2}(i+1/2,j,k+1/2)-H_{yx}^{n+1/2}(i+1/2,j,k-1/2)\Big\}$$

$$E_{yz}^{n+1}(i,j+1/2,k) = \exp\left(\frac{-4\pi\Delta t\sigma_z(i,j+1/2,k)}{\varepsilon(i,j+1/2,k)}\right)E_{yz}^{n}(i,j+1/2,k) + \frac{c\Delta t}{\Delta z\varepsilon(i,j+1/2,k)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma_z(i,j+1/2,k)}{\varepsilon(i,j+1/2,k)}\right)\left\{H_{xy}^{n+1/2}(i,j+1/2,k+1/2) - H_{xy}^{n+1/2}(i,j+1/2,k-1/2)\right. \quad (34)$$

$$\left. + H_{xz}^{n+1/2}(i,j+1/2,k+1/2) - H_{xz}^{n+1/2}(i,j+1/2,k-1/2)\right\}$$

$$E_{yx}^{n+1}(i,j+1/2,k) = \exp\left(\frac{-4\pi\Delta t\sigma_x(i,j+1/2,k)}{\varepsilon(i,j+1/2,k)}\right)E_{yx}^{n}(i,j+1/2,k) - \frac{c\Delta t}{\Delta x\varepsilon(i,j+1/2,k)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma_x(i,j+1/2,k)}{\varepsilon(i,j+1/2,k)}\right)\left\{H_{zx}^{n+1/2}(i+1/2,j+1/2,k) - H_{zx}^{n+1/2}(i-1/2,j+1/2,k)\right. \quad (35)$$

$$\left. + H_{zy}^{n+1/2}(i+1/2,j+1/2,k) - H_{zy}^{n+1/2}(i-1/2,j+1/2,k)\right\}$$

$$E_{zx}^{n+1}(i,j,k+1/2) = \exp\left(\frac{-4\pi\Delta t\sigma_x(i,j,k+1/2)}{\varepsilon(i,j,k+1/2)}\right)E_{zx}^{n}(i,j,k+1/2) + \frac{c\Delta t}{\Delta x\varepsilon(i,j,k+1/2)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma_x(i,j,k+1/2)}{\varepsilon(i,j,k+1/2)}\right)\left\{H_{yz}^{n+1/2}(i+1/2,j,k+1/2) - H_{yz}^{n+1/2}(i-1/2,j,k+1/2)\right. \quad (36)$$

$$\left. + H_{yx}^{n+1/2}(i+1/2,j,k+1/2) - H_{yx}^{n+1/2}(i-1/2,j,k+1/2)\right\}$$

$$E_{zy}^{n+1}(i,j,k+1/2) = \exp\left(\frac{-4\pi\Delta t\sigma_y(i,j,k+1/2)}{\varepsilon(i,j,k+1/2)}\right)E_{zy}^{n}(i,j,k+1/2) - \frac{c\Delta t}{\Delta y\varepsilon(i,j,k+1/2)}$$

$$\times \exp\left(\frac{-2\pi\Delta t\sigma_y(i,j,k+1/2)}{\varepsilon(i,j,k+1/2)}\right)\Big\{H_{xy}^{n+1/2}(i,j+1/2,k+1/2) - H_{xy}^{n+1/2}(i,j-1/2,k+1/2) \quad (37)$$

$$+ H_{xz}^{n+1/2}(i,j+1/2,k+1/2) - H_{xz}^{n+1/2}(i,j-1/2,k+1/2)\Big\}$$

$$H_{xy}^{n+1/2}(i,j+1/2,k+1/2) = \exp\left(-4\pi\Delta t\sigma_y^*(i,j+1/2,k+1/2)\right)H_{xy}^{n-1/2}(i,j+1/2,k+1/2)$$

$$-\frac{c\Delta t}{\Delta y}\exp\left(-2\pi\Delta t\sigma_y^*(i,j+1/2,k+1/2)\right)\Big\{E_{zx}^{n}(i,j+1,k+1/2) - E_{zx}^{n}(i,j,k+1/2) \quad (38)$$

$$+ E_{zy}^{n}(i,j+1,k+1/2) - E_{zy}^{n}(i,j,k+1/2)\Big\}$$

$$H_{xz}^{n+1/2}(i,j+1/2,k+1/2) = \exp\left(-4\pi\Delta t\sigma_z^*(i,j+1/2,k+1/2)\right)H_{xz}^{n-1/2}(i,j+1/2,k+1/2)$$

$$+\frac{c\Delta t}{\Delta z}\exp\left(-2\pi\Delta t\sigma_z^*(i,j+1/2,k+1/2)\right)\Big\{E_{yz}^{n}(i,j+1/2,k+1) - E_{yz}^{n}(i,j+1/2,k) \quad (39)$$

$$+ E_{yx}^{n}(i,j+1/2,k+1) - E_{yx}^{n}(i,j+1/2,k)\Big\}$$

$$H_{yz}^{n+1/2}(i+1/2,j,k+1/2) = \exp\left(-4\pi\Delta t\sigma_z^*(i+1/2,j,k+1/2)\right)H_{yz}^{n-1/2}(i+1/2,j,k+1/2)$$

$$-\frac{c\Delta t}{\Delta z}\exp\left(-2\pi\Delta t\sigma_z^*(i+1/2,j,k+1/2)\right)\Big\{E_{xy}^{n}(i+1/2,j,k+1) - E_{xy}^{n}(i+1/2,j,k) \quad (40)$$

$$+ E_{xz}^{n}(i+1/2,j,k+1) - E_{xz}^{n}(i+1/2,j,k)\Big\}$$

$$H_{yx}^{n+1/2}(i+1/2,j,k+1/2) = \exp\left(-4\pi\Delta t\sigma_x^*(i+1/2,j,k+1/2)\right)H_{yx}^{n-1/2}(i+1/2,j,k+1/2)$$

$$+\frac{c\Delta t}{\Delta x}\exp\left(-2\pi\Delta t\sigma_x^*(i+1/2,j,k+1/2)\right)\left\{E_{zx}^n(i+1,j,k+1/2) - E_{zx}^n(i,j,k+1/2)\right. \tag{41}$$

$$\left.+E_{zy}^n(i+1,j,k+1/2) - E_{zy}^n(i,j,k+1/2)\right\}$$

$$H_{zx}^{n+1/2}(i+1/2,j+1/2,k) = \exp\left(-4\pi\Delta t\sigma_x^*(i+1/2,j+1/2,k)\right)H_{zx}^{n-1/2}(i+1/2,j+1/2,k)$$

$$-\frac{c\Delta t}{\Delta x}\exp\left(-2\pi\Delta t\sigma_x^*(i+1/2,j+1/2,k)\right)\left\{E_{yz}^n(i+1,j+1/2,k) - E_{yz}^n(i,j+1/2,k)\right. \tag{42}$$

$$\left.+E_{yx}^n(i+1,j+1/2,k) - E_{yx}^n(i,j+1/2,k)\right\}$$

$$H_{zy}^{n+1/2}(i+1/2,j+1/2,k) = \exp\left(-4\pi\Delta t\sigma_y^*(i+1/2,j+1/2,k)\right)H_{zy}^{n-1/2}(i+1/2,j+1/2,k)$$

$$+\frac{c\Delta t}{\Delta y}\exp\left(-2\pi\Delta t\sigma_y^*(i+1/2,j+1/2,k)\right)\left\{E_{xy}^n(i+1/2,j+1,k) - E_{xy}^n(i+1/2,j,k)\right. \tag{43}$$

$$\left.+E_{xz}^n(i+1/2,j+1,k) - E_{xz}^n(i+1/2,j,k)\right\}$$

In the event a point lies in a side region of the absorption shell, then two of the σ_j quantities vanish and the remaining σ_j quantity creates absorption in a direction perpendicular to the side. In this case, by using Equations 26 and 27, Equations 32 through 43 simplify from 12 unknowns to 10 unknowns.

FIELD POINT IN ABSORPTION LAYER THAT IS PERPENDICULAR TO x DIRECTION: $\sigma_y = \sigma_z = 0$

$$
\begin{aligned}
E_x^{n+1}(i+1/2,j,k) = E_x^n(i+1/2,j,k) + \frac{c\Delta t}{\varepsilon(i+1/2,j,k)}\Big[\big\{H_{zx}^{n+1/2}(i+1/2,j+1/2,k) - \\
H_{zx}^{n+1/2}(i+1/2,j-1/2,k) + H_{zy}^{n+1/2}(i+1/2,j+1/2,k) - H_{zy}^{n+1/2}(i+1/2,j-1/2,k)\big\}\big/\Delta y \\
+\big\{H_{yz}^{n+1/2}(i+1/2,j,k+1/2) - H_{yz}^{n+1/2}(i+1/2,j,k-1/2) + H_{yx}^{n+1/2}(i+1/2,j,k+1/2) - \\
H_{yx}^{n+1/2}(i+1/2,j,k-1/2)\big\}\big/\Delta z\Big]
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
E_{yz}^{n+1}(i,j+1/2,k) = E_{yz}^n(i,j+1/2,k) + \frac{c\Delta t}{\varepsilon(i,j+1/2,k)} \\
\times\big\{H_x^{n+1/2}(i,j+1/2,k+1/2) - H_x^{n+1/2}(i,j+1/2,k-1/2)\big\}\big/\Delta z
\end{aligned}
\tag{45}
$$

$$
\begin{aligned}
E_{yx}^{n+1}(i,j+1/2,k) = \exp\left(\frac{-4\pi\Delta t\sigma_x(i,j+1/2,k)}{\varepsilon(i,j+1/2,k)}\right)E_{yx}^n(i,j+1/2,k) + \frac{c\Delta t}{\varepsilon(i,j+1/2,k)} \\
\times\exp\left(\frac{-2\pi\Delta t\sigma_x(i,j+1/2,k)}{\varepsilon(i,j+1/2,k)}\right)\big\{H_{zx}^{n+1/2}(i+1/2,j+1/2,k) - H_{zx}^{n+1/2}(i-1/2,j+1/2,k) \\
+H_{zy}^{n+1/2}(i+1/2,j+1/2,k) - H_{zy}^{n+1/2}(i-1/2,j+1/2,k)\big\}\big/\Delta x
\end{aligned}
\tag{46}
$$

$$E_{zx}^{n+1}(i,j,k+1/2) = \exp\left(\frac{-4\pi\Delta t\sigma_x(i,j,k+1/2)}{\varepsilon(i,j,k+1/2)}\right)E_{zx}^{n}(i,j,k+1/2) + \frac{c\Delta t}{\varepsilon(i,j,k+1/2)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma_x(i,j,k+1/2)}{\varepsilon(i,j,k+1/2)}\right)\left\{H_{yz}^{n+1/2}(i+1/2,j,k+1/2) - H_{yz}^{n+1/2}(i-1/2,j,k+1/2)\right. \quad (47)$$

$$\left. + H_{yx}^{n+1/2}(i+1/2,j,k+1/2) - H_{yx}^{n+1/2}(i-1/2,j,k+1/2)\right\}\Big/\Delta x$$

$$E_{zy}^{n+1}(i,j,k+1/2) = E_{zy}^{n}(i,j,k+1/2) - \frac{c\Delta t}{\varepsilon(i,j,k+1/2)}$$

$$\times\left\{H_{x}^{n+1/2}(i,j+1/2,k+1/2) - H_{x}^{n+1/2}(i,j-1/2,k+1/2)\right\}\Big/\Delta y \quad (48)$$

$$H_{x}^{n+1/2}(i,j+1/2,k+1/2) = H_{x}^{n-1/2}(i,j+1/2,k+1/2)$$

$$-c\Delta t\Big[\left\{E_{zx}^{n}(i,j+1,k+1/2) - E_{zx}^{n}(i,j,k+1/2) + E_{zy}^{n}(i,j+1,k+1/2)\right.$$

$$\left. - E_{zy}^{n}(i,j,k+1/2)\right\}\Big/\Delta y + \left\{E_{yz}^{n}(i,j+1/2,k+1) - E_{yz}^{n}(i,j+1/2,k)\right. \quad (49)$$

$$\left. + E_{yx}^{n}(i,j+1/2,k+1) - E_{yx}^{n}(i,j+1/2,k)\right\}\Big/\Delta z\Big]$$

$$H_{yz}^{n+1/2}(i+1/2,j,k+1/2) = H_{yz}^{n-1/2}(i+1/2,j,k+1/2)$$

$$-c\Delta t\left\{E_{x}^{n}(i+1/2,j,k+1) - E_{x}^{n}(i+1/2,j,k)\right\}\Big/\Delta z \quad (50)$$

$$H_{yx}^{n+1/2}(i+1/2,j,k+1/2) = \exp\left(-4\pi\Delta t\sigma_x^*(i+1/2,j,k+1/2)\right)H_{yx}^{n-1/2}(i+1/2,j,k+1/2)$$

$$+ c\Delta t \exp\left(-2\pi\Delta t\sigma_x^*(i+1/2,j,k+1/2)\right)\left\{E_{zx}^n(i+1,j,k+1/2) - E_{zx}^n(i,j,k+1/2)\right. \qquad (51)$$

$$\left. +E_{zy}^n(i+1,j,k+1/2) - E_{zy}^n(i,j,k+1/2)\right\}/\Delta x$$

$$H_{zx}^{n+1/2}(i+1/2,j+1/2,k) = \exp\left(-4\pi\Delta t\sigma_x^*(i+1/2,j+1/2,k)\right)H_{zx}^{n-1/2}(i+1/2,j+1/2,k)$$

$$- c\Delta t \exp\left(-2\pi\Delta t\sigma_x^*(i+1/2,j+1/2,k)\right)\left\{E_{yz}^n(i+1,j+1/2,k) - E_{yz}^n(i,j+1/2,k)\right. \qquad (52)$$

$$\left. +E_{yx}^n(i+1,j+1/2,k) - E_{yx}^n(i,j+1/2,k)\right\}/\Delta x$$

$$H_{zy}^{n+1/2}(i+1/2,j+1/2,k) = H_{zy}^{n-1/2}(i+1/2,j+1/2,k)$$

$$+ c\Delta t\left\{E_x^n(i+1/2,j+1,k) - E_x^n(i+1/2,j,k)\right\}/\Delta x \qquad (53)$$

FIELD POINTS IN ABSORPTION LAYER THAT IS PERPENDICULAR TO *y* DIRECTION: $\sigma_x = \sigma_z = 0$

$$E_{xy}^{n+1}(i+1/2,j,k) = \exp\left(\frac{-4\pi\Delta t\sigma_y(i+1/2,j,k)}{\varepsilon(i+1/2,j,k)}\right)E_{xy}^n(i+1/2,j,k) + \frac{c\Delta t}{\varepsilon(i+1/2,j,k)}$$

$$\times \exp\left(\frac{-2\pi\Delta t\sigma_y(i+1/2,j,k)}{\varepsilon(i+1/2,j,k)}\right)\left\{H_{zx}^{n+1/2}(i+1/2,j+1/2,k) - H_{zx}^{n+1/2}(i+1/2,j-1/2,k)\right. \qquad (54)$$

$$\left. + H_{zy}^{n+1/2}(i+1/2,j+1/2,k) - H_{zy}^{n+1/2}(i+1/2,j-1/2,k)\right\}/\Delta y$$

$$E_{xz}^{n+1}(i+1/2,j,k) = E_{xz}^{n}(i+1/2,j,k) - \frac{c\Delta t}{\varepsilon(i+1/2,j,k)}$$

$$\times\left\{H_{y}^{n+1/2}(i+1/2,j,k+1/2) - H_{y}^{n+1/2}(i+1/2,j,k-1/2)\right\}\big/\Delta z \tag{55}$$

$$E_{y}^{n+1}(i,j+1/2,k) = E_{y}^{n}(i,j+1/2,k) + \frac{c\Delta t}{\varepsilon(i,j+1/2,k)}$$

$$\times\Big[\left\{H_{xy}^{n+1/2}(i,j+1/2,k+1/2) - H_{xy}^{n+1/2}(i,j+1/2,k-1/2) + H_{xz}^{n+1/2}(i,j+1/2,k+1/2)\right.$$

$$\left.-H_{xz}^{n+1/2}(i,j+1/2,k-1/2)\right\}\big/\Delta z + \left\{H_{zx}^{n+1/2}(i+1/2,j+1/2,k) - H_{zx}^{n+1/2}(i-1/2,j+1/2,k)\right. \tag{56}$$

$$\left.+H_{zy}^{n+1/2}(i+1/2,j+1/2,k) - H_{zy}^{n+1/2}(i-1/2,j+1/2,k)\right\}\big/\Delta x\Big]$$

$$E_{zx}^{n+1}(i,j,k+1/2) = E_{zx}^{n}(i,j,k+1/2) + \frac{c\Delta t}{\varepsilon(i,j,k+1/2)}$$

$$\times\left\{H_{y}^{n+1/2}(i+1/2,j,k+1/2) - H_{y}^{n+1/2}(i-1/2,j,k+1/2)\right\}\big/\Delta x \tag{57}$$

$$E_{zy}^{n+1}(i,j,k+1/2) = \exp\left(\frac{-4\pi\Delta t\sigma_y(i,j,k+1/2)}{\varepsilon(i,j,k+1/2)}\right)E_{zy}^{n}(i,j,k+1/2) - \frac{c\Delta t}{\varepsilon(i,j,k+1/2)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma_y(i,j,k+1/2)}{\varepsilon(i,j,k+1/2)}\right)\left\{H_{xy}^{n+1/2}(i,j+1/2,k+1/2) - H_{xy}^{n+1/2}(i,j-1/2,k+1/2)\right. \tag{58}$$

$$\left.+H_{xz}^{n+1/2}(i,j+1/2,k+1/2) - H_{xz}^{n+1/2}(i,j-1/2,k+1/2)\right\}\big/\Delta y$$

$$H_{xy}^{n+1/2}(i, j+1/2, k+1/2) = \exp\left(-4\pi\Delta t\sigma_y^*(i, j+1/2, k+1/2)\right) H_{xy}^{n-1/2}(i, j+1/2, k+1/2)$$
$$- c\Delta t \exp\left(-2\pi\Delta t\sigma_y^*(i, j+1/2, k+1/2)\right)\left\{E_{zx}^n(i, j+1, k+1/2) - E_{zx}^n(i, j, k+1/2)\right. \tag{59}$$
$$\left. + E_{zy}^n(i, j+1, k+1/2) - E_{zy}^n(i, j, k+1/2)\right\}/\Delta y$$

$$H_{xz}^{n+1/2}(i, j+1/2, k+1/2) = H_{xz}^{n-1/2}(i, j+1/2, k+1/2)$$
$$+ c\Delta t\left\{E_y^n(i, j+1/2, k+1) - E_y^n(i, j+1/2, k)\right\}/\Delta z \tag{60}$$

$$H_y^{n+1/2}(i+1/2, j, k+1/2) = H_y^{n-1/2}(i+1/2, j, k+1/2)$$
$$- c\Delta t\left[\left\{E_{xy}^n(i+1/2, j, k+1) - E_{xy}^n(i+1/2, j, k) + E_{xz}^n(i+1/2, j, k+1)\right.\right.$$
$$\left. - E_{xz}^n(i+1/2, j, k)\right\}/\Delta z - \left\{E_{zx}^n(i+1, j, k+1/2) - E_{zx}^n(i, j, k+1/2)\right. \tag{61}$$
$$\left.\left. - E_{zy}^n(i+1, j, k+1/2) + E_{zy}^n(i, j, k+1/2)\right\}/\Delta x\right]$$

$$H_{zx}^{n+1/2}(i+1/2, j+1/2, k) = H_{zx}^{n-1/2}(i+1/2, j+1/2, k)$$
$$- c\Delta t\left\{E_y^n(i+1, j+1/2, k) - E_y^n(i, j+1/2, k)\right\}/\Delta x \tag{62}$$

$$H_{zy}^{n+1/2}(i+1/2,j+1/2,k)=\exp\left(-4\pi\Delta t\sigma_y^*(i+1/2,j+1/2,k)\right)H_{zy}^{n-1/2}(i+1/2,j+1/2,k)$$

$$+c\Delta t\exp\left(-2\pi\Delta t\sigma_y^*(i+1/2,j+1/2,k)\right)\left\{E_{xy}^n(i+1/2,j+1,k)-E_{xy}^n(i+1/2,j,k)\right. \qquad (63)$$

$$\left.+E_{xz}^n(i+1/2,j+1,k)-E_{xz}^n(i+1/2,j,k)\right\}/\Delta y$$

FIELD POINTS IN ABSORPTION LAYER THAT IS PERPENDICULAR TO z DIRECTION: $\sigma_x=\sigma_y=0$

$$E_{xy}^{n+1}(i+1/2,j,k)=E_{xy}^n(i+1/2,j,k)+\frac{c\Delta t}{\varepsilon(i+1/2,j,k)}$$

$$\times\left\{H_z^{n+1/2}(i+1/2,j+1/2,k)-H_z^{n+1/2}(i+1/2,j-1/2,k)\right\}/\Delta y \qquad (64)$$

$$E_{xz}^{n+1}(i+1/2,j,k)=\exp\left(\frac{-4\pi\Delta t\sigma_z(i+1/2,j,k)}{\varepsilon(i+1/2,j,k)}\right)E_{xz}^n(i+1/2,j,k)-\frac{c\Delta t}{\varepsilon(i+1/2,j,k)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma_z(i+1/2,j,k)}{\varepsilon(i+1/2,j,k)}\right)\left\{H_{yz}^{n+1/2}(i+1/2,j,k+1/2)-H_{yz}^{n+1/2}(i+1/2,j,k-1/2)\right. \qquad (65)$$

$$\left.+H_{yx}^{n+1/2}(i+1/2,j,k+1/2)-H_{yx}^{n+1/2}(i+1/2,j,k-1/2)\right\}/\Delta z$$

$$E_{yz}^{n+1}(i,j+1/2,k)=\exp\left(\frac{-4\pi\Delta t\sigma_z(i,j+1/2,k)}{\varepsilon(i,j+1/2,k)}\right)E_{yz}^n(i,j+1/2,k)+\frac{c\Delta t}{\varepsilon(i,j+1/2,k)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma_z(i,j+1/2,k)}{\varepsilon(i,j+1/2,k)}\right)\left\{H_{xy}^{n+1/2}(i,j+1/2,k+1/2)-H_{xy}^{n+1/2}(i,j+1/2,k-1/2)\right. \qquad (66)$$

$$\left.+H_{xz}^{n+1/2}(i,j+1/2,k+1/2)-H_{xz}^{n+1/2}(i,j+1/2,k-1/2)\right\}/\Delta z$$

$$E_{yx}^{n+1}(i,j+1/2,k) = E_{yx}^{n}(i,j+1/2,k) + \frac{.c\Delta t}{\varepsilon(i,j+1/2,k)}$$
$$\times\left\{H_z^{n+1/2}(i+1/2,j+1/2,k) - H_z^{n+1/2}(i-1/2,j+1/2,k)\right\}/\Delta x \quad (67)$$

$$E_z^{n+1}(i,j,k+1/2) = E_z^{n}(i,j,k+1/2) + \frac{c\Delta t}{\varepsilon(i,j,k+1/2)}$$
$$\times\Big[\Big\{H_{yz}^{n+1/2}(i+1/2,j,k+1/2) - H_{yz}^{n+1/2}(i-1/2,j,k+1/2)$$
$$+H_{yx}^{n+1/2}(i+1/2,j,k+1/2) - H_{yx}^{n+1/2}(i-1/2,j,k+1/2)\Big\}/\Delta x \quad (68)$$
$$-\Big\{H_{xy}^{n+1/2}(i,j+1/2,k+1/2) - H_{xy}^{n+1/2}(i,j-1/2,k+1/2)$$
$$+H_{xz}^{n+1/2}(i,j+1/2,k+1/2) - H_{xz}^{n+1/2}(i,j-1/2,k+1/2)\Big\}/\Delta y\Big]$$

$$H_{xy}^{n+1/2}(i,j+1/2,k+1/2) = H_{xy}^{n-1/2}(i,j+1/2,k+1/2)$$
$$-c\Delta t\left\{E_z^{n}(i,j+1,k+1/2) - E_z^{n}(i,j,k+1/2)\right\}/\Delta y \quad (69)$$

$$H_{xz}^{n+1/2}(i,j+1/2,k+1/2) = \exp\left(-4\pi\Delta t\sigma_z^{*}(i,j+1/2,k+1/2)\right)H_{xz}^{n-1/2}(i,j+1/2,k+1/2)$$
$$+c\Delta t\exp\left(-2\pi\Delta t\sigma_z^{*}(i,j+1/2,k+1/2)\right)\Big\{E_{yz}^{n}(i,j+1/2,k+1) - E_{yz}^{n}(i,j+1/2,k) \quad (70)$$
$$+E_{yx}^{n}(i,j+1/2,k+1) - E_{yx}^{n}(i,j+1/2,k)\Big\}/\Delta z$$

$$H_{yz}^{n+1/2}(i+1/2,j,k+1/2) = \exp\left(-4\pi\Delta t\sigma_z^*(i+1/2,j,k+1/2)\right)H_{yz}^{n-1/2}(i+1/2,j,k+1/2)$$

$$-c\Delta t\exp\left(-2\pi\Delta t\sigma_z^*(i+1/2,j,k+1/2)\right)\left\{E_{xy}^n(i+1/2,j,k+1) - E_{xy}^n(i+1/2,j,k)\right. \tag{71}$$

$$\left. + E_{xz}^n(i+1/2,j,k+1) - E_{xz}^n(i+1/2,j,k)\right\}\Big/\Delta z$$

$$H_{yx}^{n+1/2}(i+1/2,j,k+1/2) = H_{yx}^{n-1/2}(i+1/2,j,k+1/2)$$

$$+ c\Delta t\left\{E_z^n(i+1,j,k+1/2) - E_z^n(i,j,k+1/2)\right\}\Big/\Delta x \tag{72}$$

$$H_z^{n+1/2}(i+1/2,j+1/2,k) = H_z^{n-1/2}(i+1/2,j+1/2,k)$$

$$-c\Delta t\Big[\Big\{E_{yz}^n(i+1,j+1/2,k) - E_{yz}^n(i,j+1/2,k)$$

$$+ E_{yx}^n(i+1,j+1/2,k) - E_{yx}^n(i,j+1/2,k)\Big\}\Big/\Delta x \tag{73}$$

$$-\Big\{E_{xy}^n(i+1/2,j+1,k) - E_{xy}^n(i+1/2,j,k)$$

$$+ E_{xz}^n(i+1/2,j+1,k) - E_{xz}^n(i+1/2,j,k)\Big\}\Big/\Delta y\Big]$$

However, since most of the computational effort will be spent outside an absorption region, the Berenger anisotropic absorbing conductivities σ_j and σ_j^* ($j = x, y, z$) will be set to zero most of the time. In this case, Equations 32 through 43 will simplify. If we are outside of a Berenger absorbing region, but there are physical regions consisting of ordinary conducting media in the computational domain, rather than set σ_j and $\sigma_j^* = 0$, we can let $\sigma_j = \sigma$ and $\sigma_j^* = 0$ where σ is a spatially dependent scalar conductivity. In the event we are outside of a Berenger absorbing region and there are no conducting media ($\sigma = 0$), then the spatial dependence of ε describes the dielectric permittivity which characterizes the structure being studied (waveguide or whatever). For the case where $\sigma_j = \sigma$, and $\sigma_j^* = 0$, Equations 32 through 43 simplify to

FIELD POINT OUTSIDE ABSORPTION SHELL: $\sigma_x = \sigma_y = \sigma_z = \sigma$ AND $\sigma_x^* = \sigma_y^* = \sigma_z^* = 0$

$$E_x^{n+1}(i+1/2,j,k) = \exp\left(\frac{-4\pi\Delta t\sigma(i+1/2,j,k)}{\varepsilon(i+1/2,j,k)}\right)E_x^n(i+1/2,j,k) + \frac{c\Delta t}{\varepsilon(i+1/2,j,k)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma(i+1/2,j,k)}{\varepsilon(i+1/2,j,k)}\right)\left\{\frac{H_z^{n+1/2}(i+1/2,j+1/2,k) - H_z^{n+1/2}(i+1/2,j-1/2,k)}{\Delta y}\right. \tag{74}$$

$$\left.-\frac{H_y^{n+1/2}(i+1/2,j,k+1/2) - H_y^{n+1/2}(i+1/2,j,k-1/2)}{\Delta z}\right\}$$

$$E_y^{n+1}(i,j+1/2,k) = \exp\left(\frac{-4\pi\Delta t\sigma(i,j+1/2,k)}{\varepsilon(i,j+1/2,k)}\right)E_y^n(i,j+1/2,k) + \frac{c\Delta t}{\varepsilon(i,j+1/2,k)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma(i,j+1/2,k)}{\varepsilon(i,j+1/2,k)}\right)\left\{\frac{H_x^{n+1/2}(i,j+1/2,k+1/2) - H_x^{n+1/2}(i,j+1/2,k-1/2)}{\Delta z}\right. \tag{75}$$

$$\left.-\frac{H_z^{n+1/2}(i+1/2,j+1/2,k) - H_z^{n+1/2}(i-1/2,j+1/2,k)}{\Delta x}\right\}$$

$$E_z^{n+1}(i,j,k+1/2) = \exp\left(\frac{-4\pi\Delta t\sigma(i,j,k+1/2)}{\varepsilon(i,j,k+1/2)}\right)E_y^n(i,j,k+1/2) + \frac{c\Delta t}{\varepsilon(i,j,k+1/2)}$$

$$\times\exp\left(\frac{-2\pi\Delta t\sigma(i,j,k+1/2)}{\varepsilon(i,j,k+1/2)}\right)\left\{\frac{H_y^{n+1/2}(i+1/2,j,k+1/2) - H_y^{n+1/2}(i-1/2,j,k+1/2)}{\Delta x}\right. \tag{76}$$

$$\left.-\frac{H_x^{n+1/2}(i,j+1/2,k+1/2) - H_x^{n+1/2}(i,j-1/2,k+1/2)}{\Delta y}\right\}$$

$$H_x^{n+1/2}(i,j+1/2,k+1/2) = H_x^{n-1/2}(i,j+1/2,k+1/2)$$

$$-c\Delta t\left\{\frac{E_z^n(i,j+1,k+1/2)-E_z^n(i,j,k+1/2)}{\Delta y}-\frac{E_y^n(i,j+1/2,k+1)-E_y^n(i,j+1/2,k)}{\Delta z}\right\} \tag{77}$$

$$H_y^{n+1/2}(i+1/2,j,k+1/2) = H_y^{n-1/2}(i+1/2,j,k+1/2)$$

$$-c\Delta t\left\{\frac{E_x^n(i+1/2,j,k+1)-E_x^n(i+1/2,j,k)}{\Delta z}-\frac{E_z^n(i+1,j,k+1/2)-E_z^n(i,j,k+1/2)}{\Delta x}\right\} \tag{78}$$

$$H_z^{n+1/2}(i+1/2,j+1/2,k) = H_z^{n-1/2}(i+1/2,j+1/2,k)$$

$$-c\Delta t\left\{\frac{E_y^n(i+1,j+1/2,k)-E_y^n(i,j+1/2,k)}{\Delta x}-\frac{E_x^n(i+1/2,j+1,k)-E_x^n(i+1/2,j,k)}{\Delta y}\right\} \tag{79}$$

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